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LARGE NEUTRINO MIXING FROM SMALL QUARK AND LEPTON MIXINGS

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Abstract

We discuss the special class of models where nearly maximal neutrino mixing is produced through the see-saw mechanism, starting from only nearly diagonal matrices for charged leptons, Dirac neutrinos and Majorana right-handed neutrinos.

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1 Introduction

Recent data from Superkamiokande [1] have provided a more solid experimental basis for neutrino oscillations as an explanation of the atmospheric neutrino anomaly. In addition, also the solar neutrino deficit, observed by several experiments [2], is probably an indication of a different sort of neutrino oscillations. Neutrino oscillations imply neutrino masses. The extreme smallness of neutrino masses in comparison with quark and charged lepton masses indicates a different nature of the former, presumably linked to lepton-number violation and the Majorana nature of neutrinos. Thus neutrino masses provide a window on the very large energy scale where lepton number is violated and on Grand Unified Theories (GUTs). Experimental facts on neutrino masses and mixings could give an important feedback on the problem of quark and charged lepton masses, as all these masses are possibly related in GUTs. In particular the observation of a nearly maximal mixing angle for $\nu_\mu \rightarrow \nu_\tau$ is particularly interesting. Perhaps also solar neutrinos may occur with large mixing angle. At present solar neutrino mixings can be either large or very small, depending on which particular solution will eventually be established by the data. Large mixings in the neutrino sector are very interesting because a first guess was in favour of small mixings, in analogy to what is observed for quarks. If confirmed, single or double maximal mixings can provide an important hint on the mechanisms that generate neutrino masses. Many possible theoretical descriptions of large neutrino mixing(s) have been discussed in the literature [3]. In most models large mixings are already present at the level of Dirac and/or Majorana matrices for neutrinos. Instead, here we discuss an interesting class of models where large, possibly maximal, neutrino mixings are generated by the see-saw mechanism starting from nearly diagonal Majorana and Dirac matrices for neutrinos and charged leptons, without fine tuning or stretching small parameters into becoming large.

The experimental status of neutrino oscillations is still very preliminary. Thus, in order to be able to proceed, the theorist has to make a number of assumptions on how the data will finally look when the experimental situation will be completely clarified. Here we assume that only two distinct oscillation frequencies exist, the largest being associated with atmospheric neutrinos and the smallest with solar neutrinos. We assume that the hint of an additional frequency from the LSND experiment [4], not confirmed by the Karmen experiment [5] (but yet far from being completely excluded), will disappear. Thus we avoid the introduction of new sterile neutrino species and can deal with only the three known species of light neutrinos. We interpret the atmospheric neutrino oscillations as nearly maximal $\nu_\mu \rightarrow \nu_\tau$ oscillations, in agreement with the Chooz results [6]. The solar-neutrino oscillations correspond to the disappearance of ν_e into nearly equal fractions of ν_μ and ν_τ . A priori we are open minded about which of the three most likely solutions for solar neutrino oscillations is adopted: the two MSW solutions with small (SA) or large (LA) mixing angle, or the vacuum oscillation solution (VO).

Assuming only two frequencies, given by

$$\Delta_{sun} \propto m_2^2 - m_1^2 \quad , \quad \Delta_{atm} \propto m_3^2 - m_{1,2}^2 \quad , \quad (1)$$

there are three possible hierarchies of mass eigenvalues:

$$\begin{aligned}
\text{A} &: |m_3| \gg |m_{2,1}| \\
\text{B} &: |m_1| \sim |m_2| \gg |m_3| \\
\text{C} &: |m_1| \sim |m_2| \sim |m_3|
\end{aligned} \tag{2}$$

(in case A there is no prejudice on the m_1, m_2 relation). For B and C, different subcases are generated according to the relative sign assignments for $m_{1,2,3}$. The configurations B and C imply a very precise near degeneracy of squared masses. For example, the case C is the only one that could in principle accommodate neutrinos as hot dark matter together with solar and atmospheric neutrino oscillations. We think that it is not at all clear at the moment that a hot dark matter component is really needed [7], but this could be a reason in favour of the fully degenerate solution. However, we would need a relative splitting $|\Delta m/m| \sim \Delta m_{atm}^2/2m^2 \sim 10^{-3}-10^{-4}$ and a much smaller one for solar neutrinos, especially if explained by vacuum oscillations: $|\Delta m/m| \sim 10^{-10}-10^{-11}$. It is reasonable to assume that the Dirac neutrino matrix has a strongly hierarchical structure, with a dominant third family eigenvalue, as is the case for up quarks, down quarks and charged leptons. We consider it implausible that, starting from hierarchical Dirac matrices, we end up via the see-saw mechanism into a nearly perfect degeneracy of squared masses. Thus models with degenerate neutrinos could only be natural if the dominant contributions directly arise from non-renormalizable operators of higher dimension. However, such a precise near degeneracy, even if true at the GUT scale, could only be stable enough against renormalization group corrections when running down to low energies if it were protected by a symmetry [8].

2 A 2×2 Example

As a consequence, here we concentrate on models of type A with large effective light neutrino mass splittings and large mixings. In general large splittings correspond to small mixings because normally only close-by states are strongly mixed. In a 2 by 2 matrix context, the requirement of large splitting and large mixing leads to a condition of vanishing determinant. For example the matrix

$$m \propto \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix} \tag{3}$$

has eigenvalues 0 and $1 + x^2$ and for x of $O(1)$ the mixing is large. Thus, in the limit of neglecting small mass terms of order $m_{1,2}$, the demands of large atmospheric neutrino mixing and dominance of m_3 translate into the condition that the subdeterminant 23 of the 3 by 3 mass matrix vanishes. The problem is to show that this vanishing can be arranged in a natural way without fine tuning.

Without loss of generality we can go to a basis where both the charged lepton Dirac mass matrix m_D^l and the Majorana matrix M for the right-handed neutrinos are diagonal. In fact, after diagonalization of the charged lepton Dirac mass matrix, we still have the freedom of a

change of basis for the right-handed neutrino fields, in that the right-handed charged lepton and neutrino fields, as opposed to left-handed fields, are uncorrelated by the $SU(2) \times U(1)$ gauge symmetry. We can use this freedom to make the Majorana matrix diagonal: $M^{-1} = V^T d_M V$ with $d_M = \text{Diag}[1/M_1, 1/M_2, 1/M_3]$. Then if we parametrize the matrix $V m_D$ by z_{ab} we have:

$$(m_\nu)_{ab} \equiv (m_D^T M^{-1} m_D)_{ab} = \sum_c \frac{z_{ca} z_{cb}}{M_c}. \quad (4)$$

From this expression we see that, while we can always arrange the twelve parameters z_{ab} and M_a to arbitrarily fix the six independent matrix elements of m_ν , case A is special in that it can be approximately reproduced in two particularly simple ways, without relying on precise cancellations among different terms:

- i)* there are only two large entries in the z matrix, $|z_{c2}| \sim |z_{c3}|$, and the three eigenvalues M_a are of comparable magnitude (or, at least, with a less pronounced hierarchy than for the z matrix elements). Then, the subdeterminant 23 vanishes and one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1. This possibility was discussed in some of our previous papers [9, 10];
- ii)* one of the right-handed neutrinos is particularly light and, in first approximation, it is only coupled to μ and τ . Thus, $M_c \sim \eta$ (small) and $z_{c1} \sim 0$. In this case the 23 subdeterminant vanishes, and again one only needs the ratio $|z_{c2}/z_{c3}|$ to be close to 1. This possibility has been especially emphasized in refs. [11, 12, 13, 14].

In a 2 by 2 matrix context (in the 23 sector), a typical example of mechanism *i* is given by a Dirac matrix m_D , defined by $\bar{R} m_D L$, which takes the approximate form:

$$m_D \propto \begin{bmatrix} 0 & 0 \\ x & 1 \end{bmatrix} \quad . \quad (5)$$

This matrix has the property that for a generic Majorana matrix M one finds:

$$m_\nu \propto \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix} \quad . \quad (6)$$

The only condition on M^{-1} is that the $1/M_3$ entry be non-zero. In particular all M_i can well be of the same order. As is clear, this mechanism is based on asymmetric Dirac matrices, with, in the case of the example, a large left-handed mixing already present in the Dirac matrix. In our previous paper, ref. [10], we argued that in an $SU(5)$ GUT left-handed mixings for leptons tend to correspond to right-handed mixings for d quarks (in a basis where u quarks are diagonal). Since large right-handed mixings for quarks are not in contrast with experiment, viable GUT models can be constructed following this mechanism; they correctly reproduce the data on fermion masses and mixings.

If, for some reason, one prefers symmetric or nearly so matrices, then one can use mechanism *ii*. For example, one could want to preserve left-right symmetry at the GUT scale. Then, the

observed smallness of left-handed mixings for quarks would also demand small right-handed mixings. So we now assume that m_D is nearly diagonal (always in the basis where m_D^l and M are diagonal) with all its off-diagonal terms proportional to some small parameter ϵ . Starting from

$$m_D \propto \begin{bmatrix} \epsilon^p & x\epsilon \\ x\epsilon & 1 \end{bmatrix}, \quad M^{-1} \propto \begin{bmatrix} r_2 & 0 \\ 0 & 1 \end{bmatrix}, \quad (7)$$

where x is of $O(1)$ and $r_2 \equiv M_3/M_2$, we obtain:

$$m_\nu \propto \begin{bmatrix} \epsilon^{2p}r_2 + x^2\epsilon^2 & x\epsilon^{p+1}r_2 + x\epsilon \\ x\epsilon^{p+1}r_2 + x\epsilon & x^2\epsilon^2r_2 + 1 \end{bmatrix}. \quad (8)$$

For sufficiently small M_2 the terms in r_2 are dominant. For $p = 1, 2$, which we consider as typical cases, it is sufficient that $\epsilon^2r_2 \gg 1$. Assuming that this condition is satisfied, consider first the case with $p = 2$. We have

$$m_\nu \propto x^2\epsilon^2r_2 \begin{bmatrix} \frac{\epsilon^2}{x^2} & \frac{\epsilon}{x} \\ \frac{\epsilon}{x} & 1 \end{bmatrix}. \quad (9)$$

In this case the determinant is naturally vanishing (to the extent that the terms in r_2 are dominant), so that the mass eigenvalues are widely split. However, the mixing is nominally small: $\sin 2\theta$ is of $O(2\epsilon/x)$. It could be numerically large enough if $1/x \sim 2-3$ and ϵ is of the order of the Cabibbo angle $\epsilon \sim 0.20-0.25$. This is what we call “stretching”: the large neutrino mixing is explained in terms of a small parameter; this is not so small and can give a perhaps sufficient amount of mixing if enhanced by a possibly large coefficient. This minimalistic view was endorsed in refs. [15].

A more peculiar case is obtained for $p = 1$, which gives:

$$m_\nu \propto \epsilon^2r_2 \begin{bmatrix} 1 & x \\ x & x^2 \end{bmatrix}. \quad (10)$$

In this case the small parameter ϵ is completely factored out and for $x \sim 1$ the mixing is nearly maximal. The see-saw mechanism has created *large mixing from almost nothing*: all relevant matrices entering the see-saw mechanism are nearly diagonal. Clearly, the crucial factorization of the small parameter ϵ^2 only arises for $p = 1$, that is the light Majorana eigenvalue is coupled to ν_μ and ν_τ with comparable strength.

3 Generalization to the 3×3 Case

It is straightforward to extend the previous model to the 3 by 3 case. One simple class of examples is the following one. We start from

$$m_D = v \begin{bmatrix} \epsilon'' & \epsilon' & y\epsilon' \\ \epsilon' & \epsilon & x\epsilon \\ y\epsilon' & x\epsilon & 1 \end{bmatrix}, \quad M^{-1} = \frac{1}{\Lambda} \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix}, \quad (11)$$

where, unless otherwise stated, x and y are $O(1)$; ϵ , ϵ' and ϵ'' are independent small numbers and $r_i \equiv M_3/M_i$. We expect $\epsilon'' \ll \epsilon' \ll \epsilon \ll 1$ and, perhaps, also $r_1 \gg r_2 \gg r_3 = 1$, if the hierarchy for right-handed neutrinos follows the same pattern as for known fermions. Depending on the relative size of the ratios r_i/r_j , ϵ/ϵ' and ϵ'/ϵ'' , we can have models with dominance of any of the $r_{1,2}$.

For example, we set $x = 1$ (keeping y of $O(1)$) and assume $r_2\epsilon^2 \gg r_1\epsilon'^2, r_3$, together with $r_2\epsilon'^2 \gg r_1\epsilon''^2$ and $r_2\epsilon \gg r_1\epsilon''$. In this limit, with good accuracy we obtain:

$$m_\nu = \frac{v^2}{\Lambda} r_2 \epsilon^2 \begin{bmatrix} \frac{\epsilon'^2}{\epsilon^2} & \frac{\epsilon'}{\epsilon} & \frac{\epsilon'}{\epsilon} \\ \frac{\epsilon'}{\epsilon} & 1 + \frac{r_1 \epsilon'^2}{r_2 \epsilon^2} & 1 \\ \frac{\epsilon'}{\epsilon} & 1 & 1 + \frac{r_3}{r_2 \epsilon^2} \end{bmatrix} . \quad (12)$$

Note that, for $\epsilon\epsilon'' \leq \epsilon'^2$:

$$\text{Det}[m_\nu] = \left(\frac{v^2}{\Lambda} \right)^3 r_1 r_2 r_3 \epsilon'^4 . \quad (13)$$

Approximate eigenvalues, in units of v^2/Λ and with $m_3 > m_2 > m_1$, are given by

$$m_3 \sim 2r_2\epsilon^2, \quad m_2 \sim \frac{1}{2}r_1\epsilon'^2, \quad m_1 \sim \frac{r_3\epsilon'^2}{\epsilon^2} \quad (14)$$

for $r_1\epsilon'^2 > r_3$ or, alternatively,

$$m_3 \sim 2r_2\epsilon^2, \quad m_2 \sim \frac{1}{2}r_3, \quad m_1 \sim \frac{r_1\epsilon'^4}{\epsilon^2} \quad (15)$$

for $r_1\epsilon'^2 < r_3$. Having set $x = 1$ the atmospheric neutrino mixing is nearly maximal. The solar neutrino mixing is instead generically small in these models, being proportional to ϵ'/ϵ . Thus the SA-MSW solution is obtained. It is easy to find set of parameter values that lead to an acceptable phenomenology within these solutions. As an illustrative example we take:

$$\epsilon \sim \lambda^4, \quad \epsilon' \sim \lambda^6, \quad \epsilon'' \sim \lambda^{12}, \quad (16)$$

$$r_1 \sim \lambda^{-12}, \quad r_2 \sim \lambda^{-9}, \quad (17)$$

where $\lambda \sim \sin \theta_C$, θ_C being the Cabibbo angle. The neutrino mass matrices become

$$m_D = v \begin{bmatrix} \lambda^{12} & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{bmatrix}, \quad M = \Lambda \begin{bmatrix} \lambda^{12} & 0 & 0 \\ 0 & \lambda^9 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (18)$$

and, in units of v^2/Λ , we obtain

$$m_3 \sim 1/\lambda, \quad m_2 \sim 1, \quad m_1 \sim \lambda^4. \quad (19)$$

The solar mixing angle θ_{12} is of order λ^2 , suitable to the SA-MSW solution. Also $\theta_{13} \sim \lambda^2$.

It is also possible to arrange the parameters in eq. (11) in such a way that r_1 is dominant. Phenomenologically viable models can also be constructed in this case, always of the SA-MSW type. For instance, we can take $y = 1$ (keeping x of $O(1)$) in order to obtain a large atmospheric mixing angle, and further assume the dominance of the r_1 terms, namely: $\epsilon'^2 r_1 \gg \epsilon'^2 r_2, r_3$, with $\epsilon''^2 r_1 \gg \epsilon'^2 r_2$ and $\epsilon'' r_1 \gg \epsilon r_2, r_3$. In this case we can approximate the light neutrino mass matrix as follows:

$$m_\nu = \frac{v^2}{\Lambda} r_1 \epsilon'^2 \begin{bmatrix} \frac{\epsilon''^2}{\epsilon'^2} & \frac{\epsilon''}{\epsilon'} & \frac{\epsilon''}{\epsilon'} \\ \frac{\epsilon''}{\epsilon'} & 1 + \frac{r_2 \epsilon'^2}{r_1 \epsilon'^2} & 1 \\ \frac{\epsilon''}{\epsilon'} & 1 & 1 + \frac{r_3}{r_1 \epsilon'^2} \end{bmatrix} . \quad (20)$$

As an example we consider the following choice of parameters:

$$\epsilon \sim \lambda^4 , \quad \epsilon' \sim \lambda^6 , \quad \epsilon'' \sim \lambda^8 , \quad (21)$$

$$r_1 \sim \lambda^{-14} , \quad r_2 \sim \lambda^{-6} . \quad (22)$$

In this case we have

$$m_D = v \begin{bmatrix} \lambda^8 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{bmatrix} , \quad M = \Lambda \begin{bmatrix} \lambda^{14} & 0 & 0 \\ 0 & \lambda^6 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \quad (23)$$

In units of v^2/Λ , we find

$$m_3 \sim 1/\lambda^2 , \quad m_2 \sim 1 \gg m_1 \quad (24)$$

and $\theta_{12} \sim \lambda^2$.

An interesting feature of these textures, in connection with a possible realization within a GUT scheme, is that m_3 is much larger than v^2/Λ by construction. This means that the lepton number breaking scale Λ can be pushed up to the order of the GUT scale or even beyond. A large value of Λ , $\Lambda \geq M_{GUT}$, is naturally expected in SU(5) if right-handed neutrinos are present and in SO(10) if the grand unified group is broken down to SU(5) close to or slightly below the Planck mass. We recall that if m_3 is approximately given by v^2/Λ and $v \sim 250$ GeV, then $\Lambda \sim 10^{15}$ GeV.

Models based on symmetric matrices are directly compatible with left-right symmetry and therefore are naturally linked with SO(10). This is to be confronted with models that have large right-handed mixings for quarks, which, in SU(5), can be naturally translated into large left-handed mixings for leptons. In this connection it is interesting to observe that the proposed textures for the neutrino Dirac matrix can also work for up and down quarks. For example, the matrices

$$m_D^u \propto \begin{bmatrix} 0 & \lambda^6 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{bmatrix} , \quad m_D^d \propto \begin{bmatrix} 0 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} , \quad (25)$$

where for each entry the order of magnitude is specified in terms of $\lambda \sim \sin \theta_C$, lead to acceptable mass matrices and mixings. In fact $m_u : m_c : m_t = \lambda^8 : \lambda^4 : 1$ and $m_d : m_s : m_b = \lambda^4 : \lambda^2 : 1$. The V_{CKM} matrix receives a dominant contribution from the down sector in that the up sector angles are much smaller than the down sector ones. The same kind of texture can also be adopted in the charged lepton sector. It is simple to realize that models of this sort cannot be derived from a broken $U(1)_F$ horizontal symmetry, with a single field that breaks spontaneously $U(1)_F$. For symmetric matrices, left and right $U(1)$ charges must be equal: then, for example, if $m_{33} \sim l^{2q_3} \sim 1$ or $q_3 = 0$, then $m_{22} \sim l^{2q_2}$ is different from $m_{23} \sim l^{q_2}$ etc.

However, the symmetry requirement is not really necessary to produce a large atmospheric mixing angle from neutrino mass matrices characterized by small mixings. Indeed, by considering for simplicity the 2 by 2 case, we could equally well have started from a Dirac mass matrix of the kind:

$$m_D \propto \begin{bmatrix} \epsilon & x\epsilon \\ x' & 1 \end{bmatrix} \quad , \quad M^{-1} \propto \begin{bmatrix} r_2 & 0 \\ 0 & 1 \end{bmatrix} \quad , \quad (26)$$

with x' any number smaller than or equal to one. Once the condition $\epsilon^2 r_2 \gg 1$ is satisfied, the neutrino mass matrix of eq. (10) is obtained again, independently from x' . The generalization of this mechanism to the full 3 by 3 case is straightforward, and analogous to the one discussed above for symmetric matrices. This leaves much more freedom for model building and does not necessarily bound the realization of these schemes to left-right-symmetric scenarios.

The solar mixing angle is generically small in the class of models explicitly discussed above. However, small mixing angles in the Dirac and Majorana neutrino mass matrices do not exclude a large solar mixing angle. For instance, this is generated from the asymmetric, but nearly diagonal mass matrices:

$$m_D = v \begin{bmatrix} \lambda^6 & \lambda^6 & 0 \\ 0 & \lambda^4 & \lambda^4 \\ 0 & 0 & 1 \end{bmatrix} \quad , \quad M = \Lambda \begin{bmatrix} \lambda^{12} & 0 & 0 \\ 0 & \lambda^{10} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad . \quad (27)$$

They give rise to a light neutrino mass matrix of the kind:

$$m_\nu = \begin{bmatrix} \lambda^2 & \lambda^2 & 0 \\ \lambda^2 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \frac{v^2}{\lambda^2 \Lambda} \quad , \quad (28)$$

which is diagonalized by large θ_{12} and θ_{23} and small θ_{13} . The mass hierarchy is suitable to the large angle MSW solution.

4 Renormalization Effects

These results are stable under renormalization from the high-energy scale Λ where the mass matrices are produced down to the electroweak scale. Indeed, suppose that we start with a

supersymmetric theory whose superpotential w at the scale $\Lambda \geq M_{GUT}$ is given by:

$$w = U^c y^u Q H_u + D^c y^d Q H_d + N^c y^\nu L H_u + E^c y^e L H_d + \frac{1}{2} N^c M N^c \quad , \quad (29)$$

where

$$y^e(\Lambda) = \begin{bmatrix} 0 & 0 \\ 0 & y_\tau \end{bmatrix} \quad , \quad y^\nu(\Lambda) = \begin{bmatrix} \epsilon & x \epsilon \\ x \epsilon & 1 \end{bmatrix} \quad , \quad M(\Lambda) = \begin{bmatrix} M_2 & 0 \\ 0 & M_3 \end{bmatrix} \quad . \quad (30)$$

Here we focus on the two heaviest generations. The matrices $y^{u,d,e,\nu}$ describe the Yukawa couplings of the theory, related to the mass matrices through $m_{u,\nu} = y^{u,\nu} v_u$ and $m_{d,e} = y^{d,e} v_d$, where $v_{u,d}$ are the vacuum expectation values of the neutral scalar components in the superdoublets $H_{u,d}$. The mass scales are ordered according to $\Lambda \geq M_3 \gg M_2 \gg M_W$, M_W denoting the electroweak scale. At the scale M_2 , after integrating out the superfields N^c associated to the heavy right-handed neutrinos, the mass matrix for the light neutrinos, including the renormalization effects, is given by:

$$m_\nu(M_2) = \frac{v_u^2}{M_2} \begin{bmatrix} (y^\nu_{22})^2 & y^\nu_{22} y^\nu_{23} \\ y^\nu_{22} y^\nu_{23} & (y^\nu_{23})^2 \end{bmatrix} + O\left(\frac{1}{M_3}\right) \quad . \quad (31)$$

The couplings y^ν_{22} and y^ν_{23} are evaluated at the scale M_2 . If the running of M is computed by taking $\epsilon = 0$, then M_2 stays approximately constant when going from Λ down to M_2 . Notice that, if we neglect the terms of $O(1/M_3)$, the determinant of $m_\nu(M_2)$ vanishes, even including the renormalization effects. These effects modify the mixing angle at the scale M_2 and they depend on how the Yukawa coupling y^ν_{22} and y^ν_{23} change from Λ to M_2 . The running of the neutrino Yukawa couplings is governed by the equation:

$$\frac{dy^\nu}{dt} = \frac{1}{16\pi^2} y^\nu \left[\text{tr}(3y^{u\dagger} y^u + y^{\nu\dagger} y^\nu) - 4\pi(3\alpha_2 + \frac{3}{5}\alpha_1) + 3y^{\nu\dagger} y^\nu + y^{e\dagger} y^e \right] \quad , \quad (32)$$

from $\mu = \Lambda$ to $\mu = M_3$, with $t = \log \mu$. At M_3 , the superfield N_3^c is integrated out. This produces terms of order $1/M_3$, which are neglected in the present discussion. From $\mu = M_3$ down to $\mu = M_2$ the running of y^ν_{22} and y^ν_{23} is described by an equation formally identical to the previous one, with

$$y^\nu \equiv \begin{bmatrix} y^\nu_{22} & y^\nu_{23} \\ y^\nu_{32} & y^\nu_{33} \end{bmatrix} \rightarrow y^\nu \equiv [y^\nu_{22} \quad y^\nu_{23}] \quad . \quad (33)$$

To assess the size of the renormalization effects on y^ν_{22} and y^ν_{23} , we have integrated numerically the above equations in the approximation of constant $y^{u,e}$ and $\alpha_{1,2}$. This should provide a rough estimate of the effects. As an example we take $\Lambda = 10^{18}$ GeV, $M_3 = 10^{16}$ GeV and $M_2 = 10^8$ GeV, $\alpha_2 = (5/3)\alpha_1 = 1/24$; we choose $y_\tau = 0.6$, corresponding to the large $\tan \beta \equiv v_u/v_d$ regime, which enhances the renormalization effects of y^ν . Assuming as initial conditions $y^\nu_{22}(\Lambda) = y^\nu_{23}(\Lambda) = y^\nu_{32}(\Lambda) = 0.05$ and $y^\nu_{33}(\Lambda) = 1$, we find that y^ν_{22} and y^ν_{23} are modified by about 10% at the scale M_2 . The mixing angle, nearly maximal at the scale Λ , remains close to maximal: $\sin^2 2\theta_{23} \sim 0.98$ in the example at hand.

The running from M_2 down to the electroweak scale does not appreciably affect neither the vanishing determinant condition, nor a nearly maximal neutrino mixing angle. This can be seen

by analyzing the renormalization group equation for m_ν , which, in the one-loop approximation, is given by:

$$\frac{dm_\nu}{dt} = \frac{1}{8\pi^2} \left\{ \left[\text{tr}(3y^{u\dagger}y^u) - 4\pi(3\alpha_2 + \frac{3}{5}\alpha_1) \right] m_\nu + \frac{1}{2} \left[m_\nu y^{e\dagger}y^e + (y^{e\dagger}y^e)^T m_\nu \right] \right\} . \quad (34)$$

The solution is:

$$m_\nu(M_W) = C \begin{bmatrix} m_{\nu 22}(M_2) & m_{\nu 23}(M_2) B \\ m_{\nu 23}(M_2) B & m_{\nu 33}(M_2) B^2 \end{bmatrix} , \quad (35)$$

where

$$C = \exp \left[\int_{t_0}^t c(t') dt' \right] , \quad B = \exp \left[\int_{t_0}^t b(t') dt' \right] , \quad t_0 = \log(M_W) , \quad t = \log(M_2) \quad (36)$$

and

$$c(t) = \frac{1}{8\pi^2} \left[\text{tr}(3y^{u\dagger}y^u) - 4\pi(3\alpha_2 + \frac{3}{5}\alpha_1) \right] (t) , \quad (37)$$

$$b(t) = \frac{y_\tau^2(t)}{16\pi^2} . \quad (38)$$

We see from eq. (35) that if the determinant of m_ν vanishes at the scale M_2 , then it vanishes also at the weak scale. The mixing angle at the weak scale is given by:

$$\sin^2(2\theta)(M_W) = \frac{4m_{\nu 23}^2(M_2)B^2}{4m_{\nu 23}^2(M_2)B^2 + (m_{\nu 33}(M_2)B^2 - m_{\nu 22}(M_2))^2} . \quad (39)$$

The condition of maximal mixing at the scale M_2 is $m_{\nu 33}(M_2) = m_{\nu 22}(M_2)$. It is easy to see that, if this condition is met, then the first correction to $\sin^2(2\theta)(M_W)$ is of second order in the parameter $(B - 1)$. Numerically, we find that $\sin^2(2\theta)(M_W)$ is reduced only by a of few percent.

Summarizing, the renormalization effects do not appreciably modify the results obtained at the tree level for eigenvalues and mixing angle of the neutrino mass matrix. They manifest themselves as $O(1)$ coefficients that, for all practical purposes, can be absorbed in the definition of the classical parameters. An analogous analysis can be carried out in the 3 by 3 case, with similar results. This is in agreement with the analysis of ref. [8], as far as the case of hierarchical neutrino masses is concerned.

5 An Explicit GUT Example with Broken Flavour Symmetry

From our general discussion it is clear that some amount of correlation between the Dirac and the Majorana sectors is required in order to reproduce, at the order-of-magnitude level, the observed pattern of oscillations (assuming from now on the SA-MSW solution for solar neutrinos). In order to show that this correlation could be induced by an underlying flavour

symmetry and, at the same time, to present an explicit grand unified example of the class of textures considered here, we sketch the features of an SU(5) model with a spontaneously broken flavour symmetry based on the gauge group $U(1)_A \times U(1)_B$.

The three generations are described by Ψ_{10}^a and $\Psi_{\bar{5}}^a$, ($a = 1, 2, 3$) transforming as 10 and $\bar{5}$ of SU(5), respectively. Three more SU(5) singlets Ψ_1^a describe the right-handed neutrinos. Here we focus on the Yukawa coupling allowed by Higgs multiplets φ_5 and $\varphi_{\bar{5}}$ in the 5 and $\bar{5}$ SU(5) representations and by three flavons, φ_A^- , φ_B^- and φ_B^+ , taken, to begin with, as SU(5) singlets. The charge assignment of the various fields is summarized in table I. We assume that the flavon fields develop the following set of VEVs:

$$\langle \varphi_A^- \rangle \sim \lambda \Lambda \quad , \quad \langle \varphi_B^- \rangle \sim \lambda \Lambda \quad , \quad \langle \varphi_B^+ \rangle \sim \Lambda \quad , \quad (40)$$

where Λ denotes the cutoff of the theory and provides the mass scale suppressing the higher-dimensional operators invariant under the gauge and the flavour groups.

	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	φ_5	$\varphi_{\bar{5}}$	φ_A^-	φ_B^-	φ_B^+
SU(5)	10	$\bar{5}$	1	5	$\bar{5}$	1	1	1
$U(1)_A$	(3,2,0)	(3,1,0)	(6,3,0)	0	0	-1	0	0
$U(1)_B$	(1,0,0)	(-2,-2,-1)	(0,2,0)	0	0	0	-1	+1

Table I : Quantum numbers of matter and flavon multiplets.

In the quark sector we obtain ¹

$$m_D^u = (m_D^u)^T = \begin{bmatrix} \lambda^8 & \lambda^6 & \lambda^4 \\ \lambda^6 & \lambda^4 & \lambda^2 \\ \lambda^4 & \lambda^2 & 1 \end{bmatrix} v_u \quad , \quad m_D^d = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^4 & \lambda^3 & \lambda \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_d \quad , \quad (41)$$

from which we get the order-of-magnitude relations:

$$\begin{aligned} m_u : m_c : m_t &= \lambda^8 : \lambda^4 : 1 \\ m_d : m_s : m_b &= \lambda^6 : \lambda^3 : 1 \end{aligned} \quad (42)$$

and

$$V_{us} \sim \lambda \quad , \quad V_{ub} \sim \lambda^3 \quad , \quad V_{cb} \sim \lambda^2 \quad . \quad (43)$$

¹In eq. (41) the entries denoted by 1 in m_D^u and m_D^d are not necessarily equal. As usual, such a notation allows for $O(1)$ deviations.

Here $v_u \equiv \langle \varphi_5 \rangle$, $v_d \equiv \langle \varphi_{\bar{5}} \rangle$. Notice that V_{ub} and V_{us} are dominated by the down-quark contribution. The mass matrix for the charged leptons is the transpose of m_D^d :

$$m_D^l = (m_D^d)^T \quad (44)$$

and we find

$$m_e : m_\mu : m_\tau = \lambda^6 : \lambda^3 : 1 \quad . \quad (45)$$

At this level we obtain the well-known prediction $m_b = m_\tau$, together with the unsatisfactory relation $m_d/m_s = m_e/m_\mu$ (which, however, is an acceptable order of magnitude equality).

In the neutrino sector, the Dirac and Majorana mass matrices are given by:

$$m_D = \begin{bmatrix} \lambda^9 & \lambda^7 & \lambda^6 \\ \lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^3 & \lambda & 1 \end{bmatrix} v_u \quad , \quad M = \begin{bmatrix} \lambda^{12} & \lambda^{11} & \lambda^6 \\ \lambda^{11} & \lambda^{10} & \lambda^5 \\ \lambda^6 & \lambda^5 & 1 \end{bmatrix} \Lambda \quad . \quad (46)$$

Notice that while in the model under consideration m_D is asymmetric, it is diagonalized by small unitarity transformations, i.e. transformations that go to the identity in the limit of vanishing λ , in both the left and the right sectors. After diagonalization of the charged-lepton sector and after integrating out the heavy right-handed neutrinos, we obtain the following neutrino mass matrix in the low-energy effective theory:

$$m_\nu = \begin{bmatrix} \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^2 & 1 & 1 \\ \lambda^2 & 1 & 1 \end{bmatrix} \frac{v_u^2}{\lambda^2 \Lambda} \quad . \quad (47)$$

The $O(1)$ elements in the 23 block are produced only by the interplay of the left-handed mixing in m_D and the hierarchy in the Majorana sector. The contribution from the charged-lepton sector is subleading. An important property of m_ν is that, as a result of the see-saw mechanism and of the specific $U(1)_F$ charge assignment, the determinant of the 23 block is of $O(\lambda^2)$ (in units of $(v_u^2/\lambda^2 \Lambda)^2$). This gives rise to the desired hierarchy between atmospheric and solar frequencies. It is easy to verify that the eigenvalues of m_ν satisfy the relations:

$$m_1 : m_2 : m_3 = \lambda^8 : \lambda^2 : 1 \quad . \quad (48)$$

The atmospheric neutrino oscillations require $m_3^2 \sim 3.5 \times 10^{-3} \text{ eV}^2$. From eq. (47), taking $v_u \sim 250 \text{ GeV}$, the mass scale Λ of the heavy Majorana neutrinos turns out to be close to $2 \times 10^{16} \text{ GeV}$. The squared mass difference between the lightest states is of $O(\lambda^4) m_3^2$, appropriate to the MSW solution of the solar neutrino problem. Finally, beyond the large mixing in the 23 sector, m_ν provides a mixing angle $\theta_{12} \sim \lambda^2$ in the 12 sector, close to the range preferred by the small-angle MSW solution. In general U_{e3} is non-vanishing, of $O(\lambda^2)$. We find it encouraging that the general pattern of all that we know on fermion masses and mixings is correctly reproduced at the level of orders of magnitude.

This model is to be taken merely as an illustration, among many other possibilities, of the scenario outlined in this note: small mixing angles for the fundamental fermions giving rise to

a large mixing angle for the light neutrinos. For this reason we do not attempt to address some important problems, such as the cancellation of chiral anomalies, which we implicitly postpone to an energy scale higher than the unification scale. We have not dealt here with the problem of recovering the correct vacuum structure by minimizing the effective potential of the theory. Also, the order of magnitude description offered by this model is not intended to account for all the details of fermion masses. Even neglecting the parameters associated with the CP violating observables, some of the relevant observables are somewhat marginally reproduced. For instance, a common problem of all SU(5) unified theories, based on a minimal Higgs structure, is represented by the relation $m_D^l = (m_D^d)^T$, which, while leading to the successful $m_b = m_\tau$ boundary condition at the GUT scale, provides the wrong prediction $m_d/m_s = m_e/m_\mu$. We might overcome this problem and improve the picture [16] by assuming that the flavon fields φ_A^- , φ_B^\pm or a suitable subset of them transform in the adjoint representation of SU(5). The product $(24)^{q\bar{5}}$ (where q is a positive integer) contains both the $\bar{5}$ and the $\bar{45}$ SU(5) representations and the couplings would allow for a differentiation between the down quarks and the charged leptons. Again, since the purpose of the present model was only illustrative, we do not insist here on recovering fully realistic mass matrices. Finally, additional contributions to flavour-changing processes and to CP-violating observables are generally expected in a supersymmetric GUT. However, a reliable estimate of the corresponding effects would require a much more detailed definition of the theory than attempted here. Crucial ingredients such as the mechanism of supersymmetry breaking and its transmission to the observable sector have been ignored in the present note.

In this example we have given up a possible symmetric structure of the textures in favour of a relatively simple and economic realization of the underlying flavour symmetry. It is possible, within the SU(5) or SO(10) gauge theories, to provide slightly more elaborate examples where this symmetric structure is manifest. In the present note we have insisted on requiring both the Dirac matrices and the Majorana matrix M to be quasi-diagonal (that is with all off-diagonal entries ij suppressed with respect to the largest of the diagonal ii and jj ones). In some alternative models, all Dirac matrices are indeed quasi-diagonal, but large mixings are present in the Majorana matrix M . For instance, the idea of a large atmospheric mixing angle from intrinsically small mixings in the Dirac mass matrices has recently been advocated in the context of a $U(2)$ flavour theory [17]. In this model the Dirac neutrino mass matrix is the one displayed in eq. (11), with y of $O(\epsilon)$ (i.e. $(m_D)_{13} \ll (m_D)_{12}$) and $\epsilon'' \sim \epsilon'^2$. The $U(2)$ symmetry favours an r_1 dominance, by adequately suppressing the M_{11} element. However, to have both ν_μ and ν_τ with approximately the same coupling to the light Majorana mass eigenstate, this should possess a component of order ϵ' along the ν_3^c direction. In turn, this property is only exhibited if also the M_{33} entry is sufficiently small ². As a consequence, there are two heavy Majorana states with similar masses and there is a large mixing in the 23 Majorana sector. In this case, large mixings, which are absent from the Dirac mass matrices, arise from the Majorana sector, a physically similar possibility but technically different from that examined here.

²Technically, this suppression can be realized in the framework of an $SO(10)$ gauge group, by assuming that there are no flavour singlets that couple to $\nu_3^c \nu_3^c$.

6 Conclusions

Summarizing, in most models that describe neutrino oscillations with nearly maximal mixings, there appear large mixings in at least one of the matrices m_D , m_D^l , M (i.e. the neutrino and charged-lepton Dirac matrices and the right-handed Majorana matrix). In this note we have discussed the peculiar possibility that large neutrino mixing is only produced by the see-saw mechanism starting from all nearly diagonal matrices. Although this possibility is certainly rather special, we have shown that models of this sort can be constructed without an unrealistic amount of fine tuning and are well compatible with grand unification ideas and the related phenomenology for quark and lepton masses.

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